between s = 7 and 15 finally s = 30, could be obtained for x = 3.5 cm.; and there was even then much sputtering.

It remained to determine the effect of the speed of rotation (r/sec.) of the given electric machine on the s-values.

When x is constant, s increases nearly proportionally to the angular velocity of the plates, 1.5, 3 and 6 rotations of each plate per second being instanced. The lines pass through zero as x increases from 0 to 1 to 2 cm., the optimum spark gap. The rates at which s increases with r/\sec thus steadily increase. After this (x=2 to 4 cm.) the line merely drops, the rate of s increase with r/\sec remaining about the same. Thus, for instance, if x=4 cm. 1.5 rotations per second leave s unchanged at zero.

The effect of this on the sx graphs is apparent: the crests persist at x = 2 cm.; but that the graphs drop as a whole when r/s diminishes from 6, to 3, to 1.5 rot./sec.

3. Velocity of the Winds.—An estimate may perhaps be obtained if we use Bernoulli's equation and put $v = \sqrt{2p/\rho}$. In the graphs, figures 2, 3, 4, the s-values at cusps are very commonly s = 300 and they mount to even s = 550. Since the unit of s is about 10^{-6} atm., these data may at once be taken as pressures in dynes/cm.² Thus the velocities in the two cases are v = 770 cm./sec. frequently and v = 1000 cm./sec. in very favorable cases. These are astonishingly large values. In the small time of x/v where x = 6 cm., there is very little time for the decay of ions. The change of s with x is thus to be associated with a ring-shaped vortex of air, whose axis or line of symmetry is the needle prolonged. Hence the currents near the electrode E, when x is small, must be largely radial and outward as already instanced. The bearing of much of this, on the cathode minimum potential, will be treated later.

THE REFLECTION OF ELECTRONS FROM CRYSTALS

By Carl Eckart*

CALIFORNIA INSTITUTE OF TECHNOLOGY

Communicated May 11, 1927

The recent experiments of Davisson and Germer¹ on the reflection of electrons from a crystal of nickel have shown a strong analogy between this phenomenon and the reflection of X-rays from the same crystal; the analogy is not complete, however, and the essential differences may be summarized in the following two hypotheses.

I. A single plane of atoms reflects a very appreciable fraction of the electron wave, whereas the same plane would reflect only an inappreciable part of an X-ray wave.²

II. The wave-length of the electron wave in the interior of the crystal is not the same as the wave-length of the same wave in free space, as is the case (practically speaking) for X-rays. This corresponds to the assumption of an index of refraction for the electron wave.

The fundamental hypothesis of the wave-mechanics is that the wave-length of the electron wave in free space is $\lambda = h/mv$. Let us first consider the limiting case in which the first plane of atoms on the surface of the crystal reflects all of the wave and none of it penetrates the interior. Then the general theory of interference from a plane of atoms³ predicts that if the incident beam is normal to the surface of the crystal beams will be reflected in all azimuths perpendicular to lines of atoms in the plane. This is in agreement with figure 2. (Davisson and Germer, loc. cit.) The co-latitude θ of the beam is determined by the formula

$$d \sin \theta = n\lambda$$

where d is the distance between the corresponding lines of atoms on the surface of the crystal (not the distance between planes in the interior). The data of the last three columns of table 1 (loc. cit.) were calculated from this formula. It is also in quantitative agreement with figure 1 (loc. cit.)—a fact which appears to have been overlooked.

In the actual case, where the first plane reflects only a fraction, say α , of the incident intensity, the second will reflect $\alpha(1-\alpha)$, the third $\alpha(1-\alpha)^2$, etc.; this will not affect the previous results, but we are, in addition, enabled to calculate the intensity in the various beams. The amplitude of the reflected wave will be proportional to the absolute value of

$$\alpha \sum_{n=0}^{\infty} (1 - \alpha)^n e^{2\pi i n \gamma / \lambda'} = \alpha / [1 - 2(1 - \alpha) \cos (2\pi \gamma / \lambda') + (1 - \alpha)^2]^{1/2}$$

where $n\gamma$ is the retardation in path of the wave reflected from the *n*th plane relative to the wave reflected from the first plane, and λ' (equal to λ/μ say) is the wave-length in the interior of the crystal. If c is the distance between successive planes of atoms parallel to the surface, $\gamma = c(1 + \cos \theta)$.

This expression for the amplitude of the reflected wave has a maximum when

$$\gamma = c(1 + \cos \theta) = m\lambda/\mu$$
.

If we take $\mu=1$, this is also the position of the corresponding X-ray beam, which is not in agreement with the observed facts; but if we take $\mu=0.7$, then the formula correctly summarizes the data of table 1, columns 4, 5 and 6 (loc. cit.).

The theoretical interpretation of the constants α and μ must be the object of any more detailed treatment of the subject.

- * NATIONAL RESEARCH FELLOW.
- ¹ Nature, 119, 558 (1927).
- ² This hypothesis was suggested to the author in conversation with Dr. F. Zwicky.
- ³ Sommerfeld, "Atombau," 4th ed., p. 222.

COUPLING OF ELECTRONIC AND NUCLEAR MOTIONS IN DIATOMIC MOLECULES

By Edward Uhler Condon*

GÖTTINGEN, GERMANY

Communicated March 19, 1927

While for many important problems the quantum mechanical coupling between the motion of the nuclei and of the electrons in a diatomic molecule may be ignored, there are others in which this coupling is the main factor. Among the various phenomena which are conditioned by this coupling and which have recently attracted attention the following may be mentioned:

- 1. The mechanism of the dissociation of molecules by light absorption. This question has been discussed by Franck who, with his co-workers, is also doing important experimental work in this field.¹
- 2. The mechanism of dissociation of molecules as a primary consequence of excitation by electron collision. The ideas of Franck on (1) have here been applied by Birge and Sponer² to a discussion of the experimental results of Hogness and Lunn.
- 3. The relative transition probabilities for the various vibrational transitions associated with a single electron transition in the emission or absorption of light by a molecule; i.e., the problem of intensity distribution in electronic band systems. This question has been treated recently by the writer.³
- 4. The mechanism of the process whereby molecules may be excited to vibration levels by electron collision without simultaneous electronic excitation. That this process actually occurs is proved by the recent experiments of Harries, carried out in Franck's laboratory.⁴

All of these phenomena permit of being understood in terms of the new quantum mechanics, at least in a qualitative way, as it is the purpose of this note to show.

The complete quantum-mechanical problem of a diatomic molecule, from the Schrödinger point of view, calls for the solution of a partial differential equation in 3N+6 independent variables, if N is the number of electrons in the molecule. The problem is distinguished from the prob-